

Experimental and Theoretical Investigation of the Geometry and Vibrational Frequencies of 1,2,3-Triazole, 1,2,4-Triazole, and Tetrazole Anions

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Received: June 16, 1997; In Final Form: September 15, 1997[⊗]

Optimized geometries, vibrational frequencies, scale factors, and IR absorption intensities were calculated for the 1,2,3-triazole anion (**1**) at the B3LYP, LDA, BLYP, BVWN, HF, and MP2 levels of theory using the 6-31G** (BS1), 6-31++G** (BS2), cc-pVDZ (BS3) and aug-cc-pVDZ (BS4) basis sets and for 1,2,4-triazole (**2**) and tetrazole (**3**) anions at the same levels of theory using the BS1 and BS2 basis sets only. The calculations were also carried out at the AM1, PM3 and MNDO semiempirical levels. For comparison with the calculated results, the IR and Raman spectra of **2** and **3** were measured, while for **1** the experimental assignment reported in the literature was used. Measurement of the spectra of **2** and **3** resulted in a complete assignment of the fundamental vibrations of both anions except for few bands corresponding only to the C–H stretching and out-of-plane modes. The force fields were scaled with respect to the experimental frequencies using one-scale-factor (1SF) scaling. The agreement between the calculated and experimental frequencies was excellent, especially with the B3LYP method, scaled and unscaled. The agreement between the experimental and calculated IR absorption intensities was generally satisfactory. The calculated scale factors for the studied molecules were larger than commonly calculated for other neutral molecules. The basis set effect on the calculated results was generally small. Differences and similarities between the calculated frequencies and scale factors by the different methods were rationalized in terms of the optimized geometries.

Introduction

Recently, the calculated vibrational frequencies and IR absorption intensities using the density functional theory (DFT) have drawn much attention that the use of the DFT method for the vibrational analysis is now routine. This wide interest in the vibrational frequencies calculated by the DFT method is due to its excellent accuracy to computational expense ratio.¹ On the other hand, the existence of different density functionals led to many reports which compare between the vibrational frequencies^{1–11} and other molecular properties¹² obtained by these functionals. Most of these publications conclude that the B3LYP functional¹ is the preferred method for the vibrational analysis.

In a previous publication,² a thorough comparison among the optimized geometries, vibrational frequencies, and force field scale factors obtained by the B3LYP, LDA, BVWN, and BLYP functionals, as well as the HF and MP2 methods, was made. The similarities and differences between the calculated frequencies and scale factors obtained by these methods were rationalized in terms of the optimized geometries. It was concluded

that the B3LYP and LDA functionals behave similarly and that the BLYP and BVWN functionals behave similarly and different from the B3LYP and LDA functionals. One advantage of the B3LYP functional is that the calculated vibrational frequencies are generally overestimated and the overestimation decreases with decrease of frequency, a behavior which is similar to the MP2 and HF calculated frequencies.^{2,14–16} This property is not found in the BLYP and BVWN functionals. Consequently, scaling the B3LYP force field improves the agreement between the calculated and experimental frequencies where it remained unchanged for the BLYP and BVWN functionals.^{2,9} The calculated frequencies for the C–H stretching modes by the LDA functional are not as overestimated as those obtained by the B3LYP functional and some bands, mainly out-of-plane, are underestimated. Consequently, scaling the LDA force field did not improve the average error as with the B3LYP method, but the average error was better than that obtained with the BLYP and BVWN methods. It was also concluded that the basis set effect, as usually known,^{2,6,7} is small.

Yet with the growing interest in the DFT method, no comprehensive comparison between the different density functionals, according to the best of our knowledge, was reported for the vibrational analysis of anions which is the purpose of the current publication. There are though some reports of the

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[⊗] Abstract published in *Advance ACS Abstracts*, December 15, 1997.

calculated frequencies by the DFT method for ionic species.^{17–22} The most important of these is the calculated frequencies of N,N,N',N' -tetramethyl-*p*-phenylenediamine radical cation.²² In that publication the author reported scale factors larger than those obtained for neutral molecules. Also Schaefer and co-workers in a few recent publications^{18,23} indicated that the DFT method is applicable to anions, contrary to what was previously proposed. This was demonstrated by calculating the electron affinities and vibrational frequencies of SiF_n^- ($n = 1–5$) by different DFT methods.

In the present publication, we report the vibrational spectra of **2** and **3**. In addition we compare the optimized geometries, vibrational frequencies, force field scale factors, and IR absorption intensities of **1**, **2**, and **3** at the B3LYP, LDA, BVWN, BLYP, HF, MP2, AM1, PM3, and MNDO levels of theory. To investigate the basis set effect, the calculations of **1** were done using four moderate size basis sets, with and without diffuse functions. The diffuse functions are known to be necessary for calculating the chemical properties of anions. The basis sets used are the BS1, BS2, BS3 and BS4 basis sets. For **2** and **3** the calculations were performed using the BS1 and BS2 basis sets only.

The experimental fundamental vibrations used for comparison with the calculated frequencies of **1** are those reported by Törnkvist et al.²⁴ The authors also reported the calculated vibrational frequencies and IR absorption intensities of **1** with the MNDO, HF/4-21G(N*) and MP2/6-31G* methods. For **2** and **3**, the experimental vibrational frequencies reported in the current work were used for comparison with the calculated results. Although there is an experimental IR spectrum of sodium tetrazolate monohydrate previously reported,²⁵ the reported frequencies were not comparable to the calculated frequencies reported in the present publication, while there is no experimental spectrum of **2**, we know of, previously reported.

Experimental Details

Both the 1,2,4-triazole and tetrazole samples were obtained from Sigma Chemical Co. Tetrazole was used without further purification. 1,2,4-triazole was recrystallized by dissolving in hot water (60 °C) and then slowly cooling to room temperature. Infrared and Raman spectra of the 1,2,4-triazole and tetrazole anions were measured in aqueous 0.5 M NaOH solution at pH 12. Infrared spectra were also remeasured in NaOD solution in D₂O at pD 12. Powder samples were directly added to 0.5 M NaOH (NaOD) solution until the resulting pH (pD) reached 12. Corning 145 pH meter was used to determine pH. The infrared spectra of these solutions were recorded on an FTS-40 Biorad Digilab FTIR spectrometer at 4 cm⁻¹ resolution. A homemade cell consisting of BaF₂ windows separated by a 15- μm Teflon spacer was used. Infrared and Raman spectra were corrected by subtracting identically obtained spectra of just the solvent. Raman spectra were measured with 90° axial excitation/transverse viewing geometry using a spectrometer assembled in house.²⁶ This spectrometer consists of an Ar ion laser (Coherent Innova 300), a 0.64-m spectrograph (ISA J-Y 640), and an intensified diode array detector (PAR 1420 with an OMA III controller). The excitation wavelength was the 488.0-nm argon line. An interference filter was used to remove the plasma lines from the excitation beam. After collimation, scattered light was passed through a holographic notch filter (Kaiser) and focused onto the entrance slit of the spectrograph. A slit width of 100 μm and a 600 groove/mm grating were used to yield a resolution of 6–8 cm⁻¹. An 1800 groove/mm grating was used to remeasure the Raman spectrum of just the out-of-

plane bending region (600–900 cm⁻¹). For polarized measurements, a Polaroid analyzer followed by a depolarizer was inserted into the scattered light beam. Depolarization ratios were calculated from peak areas obtained by fitting the peaks with Gaussian/Lorentzian functions.

Computational Details

The optimized geometries, vibrational frequencies and IR absorption intensities were calculated for **1**, **2**, and **3** under the C_{2v} symmetry constraint. The calculations were performed at the B3LYP,¹ LDA,²⁷ BLYP,²⁸ BVWN, HF,²⁹ and MP2³⁰ levels of theory using the BS1,³¹ BS2,³¹ BS3,³² and BS4³² basis sets for **1** and using the BS1 and BS2 basis sets for **2** and **3**. The MP2 calculations were performed keeping the 1s orbitals of the heavier centers frozen (MP2/fc). The number of basis functions of **1** with the BS1, BS2, BS3, and BS4 basis sets are 85, 107, 80, and 133, respectively. For **2** and **3**, the number of basis functions are 80/101 and 104/130, respectively, with the BS1/BS2 basis sets. In addition, the calculations were performed at the AM1,³³ PM3³⁴ and MNDO³⁵ semiempirical levels of theory.

All calculations were performed using the Gaussian94³⁶ program. The force fields were calculated by the analytical method. Conversion of the Cartesian coordinate force fields to internal coordinate force fields, scaling of the internal coordinate force fields and calculation of the frequencies is as described elsewhere.^{2,14,37} Under the C_{2v} symmetry constraint, the 15 fundamental vibrations of **1** and **2** are classified as 6A₁, 5B₂, 2A₂, and 2B₁. For **3** under the C_{2v} symmetry, the 12 fundamental vibrations are classified as 5A₁, 4B₂, A₂, and 2B₁.

Results and Discussion

Optimized Geometries. The calculated optimized geometries of **1**, **2**, and **3** are given in Tables 1, 2, and 3, respectively. The atom numbering is shown in Figure 1. First, we mention a summary about the predicted geometries by different methods. It is known that the HF-predicted bond lengths are underestimated,⁵ the MP2 predicted bond lengths are usually overestimated,^{5,38} and the DFT-predicted bond lengths are often overestimated.^{2,5,6,8} The bond angles predicted by the different DFT methods show little dependence on the functional used.² The BLYP- and LDA-functionals-predicted bond lengths are 0.01–0.02 Å too long.^{2,5,6} The B3LYP-predicted geometry has the best agreement with the experimental geometry.^{2,8,14–16}

Our previous results for five-membered heterocyclic molecules^{2,14–16} show not only that the MP2-predicted geometry is only slightly worse than the B3LYP-predicted geometry but also that the B3LYP-predicted geometry has poor prediction of some bond lengths and angles, as the C–S bond length and NSN bond angle, which is not observed by the MP2-predicted geometry. The LDA-predicted geometry is worse than the MP2-predicted geometry. The BVWN- and BLYP-predicted geometries are close to each other, and both are worse than the LDA-predicted geometry.

Several observations can be made about the results in Table 1 for the optimized geometry of **1**. The bond lengths and angles predicted by the BLYP and BVWN methods by the four basis sets used in this study are very close to each other. With the exception of the C–H bond length, the maximum difference between the bond lengths predicted by the BLYP and BVWN methods is not more than 0.003 Å and those of the bond angles is not more than 0.3°. The predicted C–H bond length by the BVWN method is shorter by about 0.006 Å than that predicted by the BLYP method. The bond lengths and angles

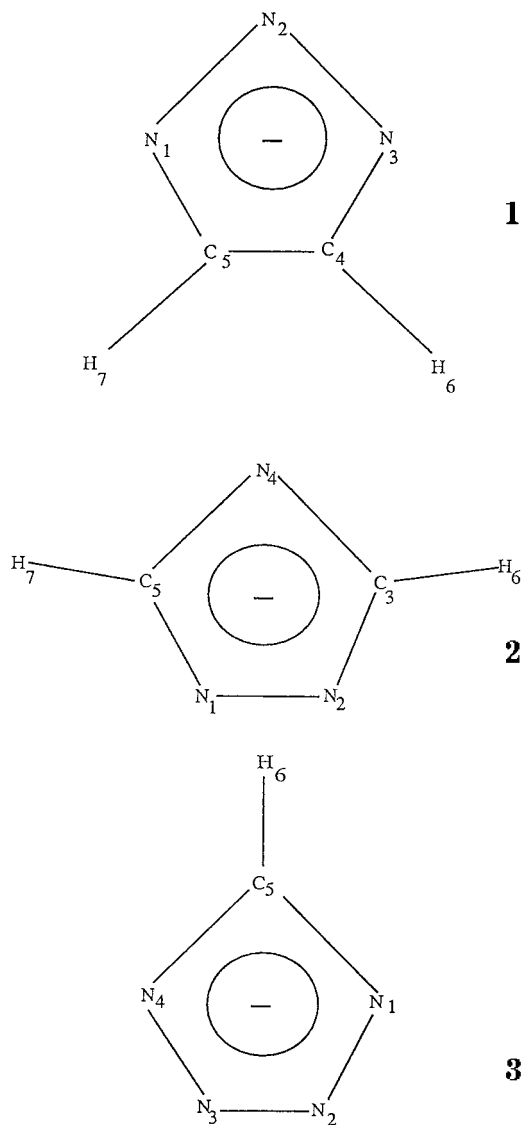


Figure 1. Atom numbering of 1,2,3-triazole (**1**), 1,2,4-triazole (**2**), and tetrazole(**3**).

predicted by the B3LYP and LDA methods are also very close to each other, but the difference between the bond lengths is larger than that between the BLYP and BVWN predicted bond lengths. Except for the C–H bond length, the B3LYP-predicted bond lengths are longer by about 0.006 Å than the LDA-predicted bond lengths. The C–H bond length predicted by the B3LYP method is shorter by about 0.01 Å than that predicted by the LDA method. The bond angles differ by not more than 0.2° in most of the cases. The B3LYP- and LDA-predicted bond lengths, as was reported for furan and thiophene,² are shorter than the BLYP- and BVWN-predicted bond lengths. A difference of about 0.01–0.03 Å is found between the LDA- and BLYP-predicted bond lengths. The bond angles are very similar.

The MP2-predicted geometry is closer to the BLYP- and BVWN-predicted geometries than the B3LYP- and LDA-predicted geometries, contrary to what was found for furan and thiophene.² The MP2-predicted bond lengths are about 0.01 Å shorter than the BVWN-predicted bond lengths, and the bond angles differ by not more than 0.2°. Generally, the MP2-predicted geometry is in between the LDA- and BLYP-predicted geometries. The bond lengths predicted by the BS3 and BS4 basis sets are longer by about 0.005 Å than those predicted by the BS1 and BS2 basis sets, and the bond angles are very similar.

Except for the C–H bond length, the MP2/fc/BS1-predicted geometry is very close to the MP2/full/6-31G*-predicted geometry reported by Törnkvist et al.²⁴ to within 0.002 Å for the bond lengths and 0.1 degree for the bond angles. The predicted C–H bond length by Törnkvist et al.²⁴ is longer than that predicted in this work using the BS1 basis set by 0.005 Å. The HF-predicted geometries have the shortest bond lengths. The bond lengths predicted by the BS3 and BS4 basis sets, as with the MP2-predicted geometry, are about 0.005 Å longer than those predicted by the BS1 and BS2 basis sets, and the bond angles are very similar.

The effect of the basis set on the predicted geometries by the different DFT methods is, as expected, small. The difference in most of the cases is not more than 0.003 Å for the bond lengths and 0.3° for the bond angles from the BS1-predicted geometries. The noticeable difference is for the C–H bond length by the BS3 basis set which is predicted to be 0.007 Å longer than that predicted by the BS1 basis set by the four DFT methods.

Similar observations for **1** are found for **2** and **3** with the exception that for **2** and **3** the N₁–N₂ B3LYP-predicted bond length is longer by about 0.01 Å than that predicted by the LDA method; the MP2 predicted geometry is closer to the B3LYP and LDA geometries than the BLYP and BVWN geometries. Also, the effect of the basis set effect on the predicted geometries is very small except with the DFT functional; the CNC bond angle of **2** using the BS2 basis set is bigger by about 1° than that predicted by the BS1 basis set, and the NCN bond angle of **2** and **3** with the BS2 basis set is smaller by about 1° than that with the BS1 basis set.

The predicted N–N bond lengths by the AM1, PM3, and MNDO methods are in most of the cases shorter even than those predicted at the HF level. The C–C bond length of **1** and the N–C bond lengths of all anions are the longest among all methods used. The C–H bond length and the bond angles are comparable to those obtained by the other methods. The only available experimental geometry is a X-ray geometry measured for sodium tetrazolate monohydrate.³⁹ Due to the limitations of the X-ray method, it not expected that this experimental geometry to be of accuracy comparable to that obtained by the microwave or electron diffraction methods. However, the agreement between the calculated and experimental geometry of **3** is qualitatively satisfactory.

Vibrational Frequencies. The measured IR and Raman spectra of **2** and **3** are given in Figures 2 and 3, respectively. The calculated and experimental frequencies of **1**, **2**, and **3** are given in Tables 4, 5, and 6, respectively. Appended in Tables 4–6 are the average errors with respect to the scaling assignment (SA). In this assignment, the experimental frequencies were assigned to the calculated frequencies in decreasing frequency order according to the Raman polarization data, whenever available. The IR frequencies, whenever available, rather than the Raman frequencies were used in the SA. All of the out-of-plane modes were excluded from the SA since some are observed only in the KBr pellet spectrum, as for **1**, or observed as a very weak features in the Raman spectra for which no reliable polarization data is available. Bands at 3114 and 3103 cm⁻¹ of **2** were also observed as very weak bands and were excluded from the SA. Also a very weak band of **1** observed at 1230 cm⁻¹ was excluded from the SA since this band had a high deviation from the corresponding calculated frequency, before and after scaling, by all methods. This band could be an impurity from the reacting species. All the other fundamental vibrations of **1**, **2**, and **3** were used in the SA. In the following

TABLE 1: Equilibrium Geometry for 1^{a,b}

| coordinate | B3LYP | | | | LDA | | | | BLYP | | | |
|--|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| | BS1 | BS2 | BS3 | BS4 | BS1 | BS2 | BS3 | BS4 | BS1 | BS2 | BS3 | BS4 |
| N ₁ -N ₂ | 1.346 | 1.344 | 1.343 | 1.345 | 1.338 | 1.335 | 1.335 | 1.336 | 1.369 | 1.367 | 1.366 | 1.367 |
| N ₁ -C ₅ | 1.355 | 1.356 | 1.357 | 1.357 | 1.349 | 1.351 | 1.351 | 1.351 | 1.369 | 1.371 | 1.370 | 1.371 |
| C ₄ -C ₅ | 1.391 | 1.394 | 1.395 | 1.395 | 1.387 | 1.389 | 1.391 | 1.391 | 1.402 | 1.404 | 1.406 | 1.406 |
| C ₄ -H ₆ | 1.086 | 1.084 | 1.093 | 1.089 | 1.095 | 1.093 | 1.102 | 1.098 | 1.094 | 1.092 | 1.102 | 1.096 |
| N ₁ N ₂ N ₃ | 111.1 | 110.9 | 111.3 | 111.0 | 111.2 | 111.0 | 111.5 | 111.1 | 111.0 | 110.8 | 111.2 | 110.9 |
| N ₂ N ₃ C ₄ | 106.6 | 106.9 | 106.7 | 106.9 | 106.7 | 107.0 | 106.7 | 107.0 | 106.3 | 106.7 | 106.4 | 106.6 |
| N ₃ C ₄ C ₅ | 107.9 | 107.7 | 107.7 | 107.6 | 107.7 | 107.5 | 107.8 | 107.5 | 108.2 | 107.9 | 108.0 | 108.0 |
| N ₃ C ₄ H ₆ | 121.9 | 121.7 | 121.8 | 121.8 | 121.8 | 121.6 | 121.8 | 121.7 | 121.7 | 121.5 | 121.6 | 121.6 |
| C ₅ C ₄ H ₆ | 130.2 | 130.6 | 130.5 | 130.6 | 130.5 | 130.9 | 130.4 | 130.8 | 129.9 | 130.6 | 130.4 | 130.4 |

| coordinate | BVWN | | | | HF | | | | MP2 | | | | AM1 | PM3 | MNDO |
|--|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| | BS1 | BS2 | BS3 | BS4 | BS1 | BS2 | BS3 | BS4 | BS1 | BS2 | BS3 | BS4 | | | |
| N ₁ -N ₂ | 1.369 | 1.367 | 1.366 | 1.367 | 1.309 | 1.308 | 1.307 | 1.309 | 1.361 | 1.362 | 1.358 | 1.367 | 1.309 | 1.315 | 1.303 |
| N ₁ -C ₅ | 1.368 | 1.369 | 1.369 | 1.370 | 1.334 | 1.336 | 1.336 | 1.339 | 1.360 | 1.363 | 1.365 | 1.369 | 1.386 | 1.385 | 1.381 |
| C ₄ -C ₅ | 1.400 | 1.402 | 1.404 | 1.403 | 1.377 | 1.380 | 1.381 | 1.382 | 1.389 | 1.392 | 1.401 | 1.403 | 1.429 | 1.396 | 1.412 |
| C ₄ -H ₆ | 1.089 | 1.086 | 1.096 | 1.091 | 1.075 | 1.073 | 1.082 | 1.079 | 1.092 | 1.081 | 1.095 | 1.092 | 1.082 | 1.084 | 1.079 |
| N ₁ N ₂ N ₃ | 110.9 | 110.7 | 111.2 | 110.9 | 111.5 | 111.4 | 111.7 | 111.6 | 110.9 | 110.6 | 111.4 | 110.9 | 113.9 | 110.4 | 112.9 |
| N ₂ N ₃ C ₄ | 106.3 | 106.6 | 106.3 | 106.6 | 107.1 | 107.3 | 107.1 | 107.2 | 106.3 | 106.6 | 106.3 | 106.5 | 107.6 | 108.8 | 107.6 |
| N ₃ C ₄ C ₅ | 108.3 | 108.1 | 108.1 | 108.0 | 107.1 | 107.0 | 107.1 | 107.0 | 108.3 | 108.1 | 108.0 | 108.1 | 105.5 | 106.0 | 106.0 |
| N ₃ C ₄ H ₆ | 121.7 | 121.6 | 121.7 | 121.7 | 122.2 | 122.2 | 122.2 | 122.2 | 121.6 | 121.4 | 121.7 | 121.5 | 123.9 | 122.9 | 123.1 |
| C ₅ C ₄ H ₆ | 130.0 | 130.3 | 130.2 | 130.3 | 130.6 | 130.8 | 130.7 | 130.8 | 130.1 | 130.5 | 130.3 | 130.4 | 130.6 | 131.1 | 130.9 |

^a Bond lengths in angstroms and angles in degrees. ^b BS1, BS2, BS3, and BS4 stand for 6-31G**, 6-31++G**, cc-pVDZ, and aug-cc-pVDZ basis sets, respectively.

TABLE 2: Equilibrium Geometry for 2^{a,b}

| coordinate | B3LYP | | LDA | | BLYP | | BVWN | | HF | | MP2 | | AM1 | PM3 | MNDO |
|--|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| | BS1 | BS2 | BS1 | BS2 | BS1 | BS2 | BS1 | BS2 | BS1 | BS2 | BS1 | BS2 | | | |
| N ₁ -N ₂ | 1.380 | 1.377 | 1.364 | 1.360 | 1.404 | 1.400 | 1.405 | 1.402 | 1.353 | 1.351 | 1.382 | 1.383 | 1.297 | 1.330 | 1.299 |
| N ₂ -C ₃ | 1.338 | 1.339 | 1.337 | 1.338 | 1.353 | 1.354 | 1.351 | 1.352 | 1.310 | 1.312 | 1.348 | 1.351 | 1.396 | 1.369 | 1.378 |
| C ₃ -N ₄ | 1.356 | 1.356 | 1.349 | 1.349 | 1.370 | 1.370 | 1.369 | 1.369 | 1.337 | 1.338 | 1.358 | 1.360 | 1.381 | 1.376 | 1.370 |
| C ₃ -H ₆ | 1.089 | 1.086 | 1.098 | 1.096 | 1.097 | 1.094 | 1.091 | 1.088 | 1.078 | 1.075 | 1.084 | 1.082 | 1.087 | 1.087 | 1.082 |
| N ₁ N ₂ C ₃ | 110.4 | 110.2 | 110.6 | 110.6 | 110.2 | 110.0 | 110.0 | 109.8 | 111.0 | 110.8 | 110.4 | 110.2 | 113.6 | 111.2 | 113.0 |
| N ₂ C ₃ N ₄ | 115.6 | 114.9 | 115.5 | 114.7 | 116.0 | 115.2 | 116.1 | 115.3 | 115.0 | 114.6 | 115.7 | 115.1 | 111.6 | 108.7 | 111.3 |
| C ₃ N ₄ C ₅ | 99.2 | 100.0 | 99.2 | 100.0 | 98.9 | 99.8 | 98.9 | 99.8 | 99.5 | 100.0 | 99.1 | 99.8 | 101.6 | 105.7 | 102.2 |
| N ₂ C ₃ H ₆ | 121.9 | 122.1 | 121.9 | 122.1 | 121.7 | 122.0 | 121.8 | 122.0 | 122.1 | 122.3 | 121.7 | 121.8 | 124.1 | 126.1 | 124.4 |
| N ₄ C ₃ H ₆ | 122.5 | 123.0 | 122.7 | 123.3 | 122.2 | 122.9 | 122.2 | 122.7 | 122.9 | 123.2 | 122.6 | 123.1 | 124.3 | 125.2 | 124.4 |

^{a,b}See corresponding footnotes in Table 1.

TABLE 3: Equilibrium Geometry for 3^{a,b}

| coordinate | B3LYP | | LDA | | BLYP | | BVWN | | HF | | MP2 | | AM1 | PM3 | MNDO | exptl ^c |
|--|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|--------------------|
| | BS1 | BS2 | BS1 | BS2 | BS1 | BS2 | BS1 | BS2 | BS1 | BS2 | BS1 | BS2 | | | | |
| N ₁ -N ₂ | 1.353 | 1.352 | 1.343 | 1.341 | 1.376 | 1.376 | 1.376 | 1.374 | 1.322 | 1.321 | 1.360 | 1.361 | 1.306 | 1.321 | 1.303 | 1.348 |
| N ₂ -N ₃ | 1.325 | 1.324 | 1.322 | 1.321 | 1.346 | 1.345 | 1.345 | 1.344 | 1.283 | 1.283 | 1.348 | 1.350 | 1.320 | 1.305 | 1.307 | 1.310 |
| N ₁ -C ₅ | 1.340 | 1.341 | 1.337 | 1.337 | 1.354 | 1.355 | 1.353 | 1.354 | 1.317 | 1.318 | 1.346 | 1.348 | 1.391 | 1.370 | 1.377 | 1.329 |
| C ₅ -H ₆ | 1.086 | 1.083 | 1.094 | 1.093 | 1.094 | 1.091 | 1.088 | 1.085 | 1.074 | 1.072 | 1.081 | 1.079 | 1.085 | 1.086 | 1.079 | 0.911 |
| N ₁ N ₂ N ₃ | 106.2 | 106.4 | 106.4 | 106.8 | 105.2 | 106.0 | 105.2 | 105.6 | 107.8 | 107.8 | 105.2 | 105.6 | 113.8 | 113.0 | 113.6 | 109.5 |
| N ₁ C ₅ N ₄ | 113.4 | 112.9 | 113.2 | 112.6 | 114.1 | 113.4 | 114.1 | 113.5 | 112.4 | 112.1 | 113.8 | 113.3 | 108.0 | 106.0 | 107.8 | 112.5 |
| N ₂ N ₃ C ₅ | 103.5 | 103.9 | 103.6 | 104.0 | 103.3 | 103.6 | 103.3 | 103.7 | 103.7 | 104.0 | 103.6 | 103.9 | 105.1 | 107.5 | 105.4 | 104.3 |
| N ₁ C ₅ H ₆ | 123.3 | 123.6 | 123.4 | 123.7 | 123.0 | 123.3 | 122.9 | 123.2 | 123.8 | 124.0 | 123.6 | 123.3 | 126.0 | 127.0 | 126.1 | 123.8 |

^{a,b}See corresponding footnotes in Table 1. ^c Reference 39.

discussion, since the out-of-plane modes were excluded from the SA and the great anharmonicity of the C-H modes, referring to the average error refers to the in-plane modes rather than the total average error unless otherwise indicated.

The calculated frequencies of **1**, **2**, and **3** indicate that, among the levels of theory used, the calculated vibrational frequencies by the B3LYP method have the lowest average error. This is compared to the highest average error obtained by the B3LYP method for furan and thiophene,² with the exception of the HF and MP2 methods. The calculated frequencies of the C-H stretching modes by the B3LYP method have the highest average error among the DFT functionals used. Although the LDA-predicted bond lengths are slightly shorter than the

B3LYP-predicted bond lengths, most of the LDA-calculated frequencies are at lower frequencies than those calculated by B3LYP method and some are underestimated compared to the experimental frequencies. Consequently, the average error obtained by the LDA method is larger than that obtained by the B3LYP method using the same basis set. The calculated C-H stretching frequencies by the LDA method are predicted at lower frequencies and in better agreement with the experimental frequencies than those predicted by the B3LYP method. This is in agreement with the longer C-H bond length predicted by the LDA method than that predicted by the B3LYP method.

Noting that the BLYP- and BVWN-predicted geometries are close to each other and that the predicted bond lengths by both

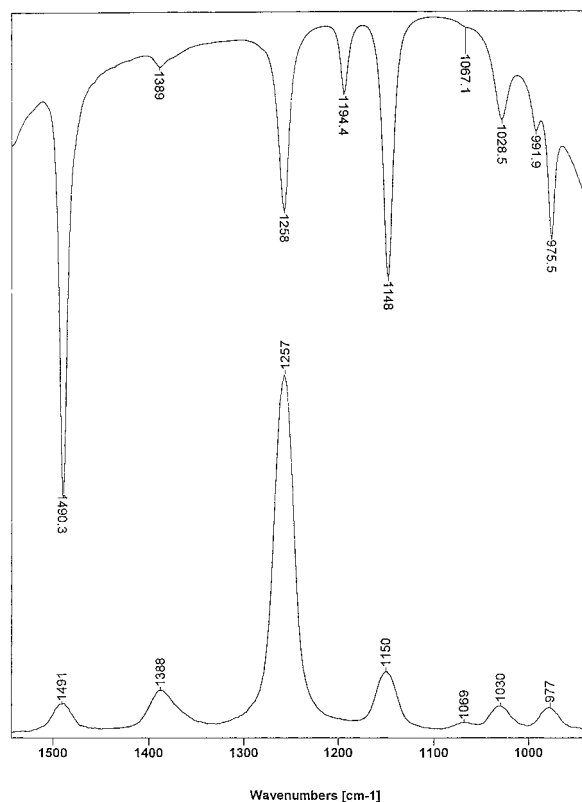


Figure 2. FTIR (top) and FT Raman (bottom) spectra of 1,2,4-triazole (2) anion.

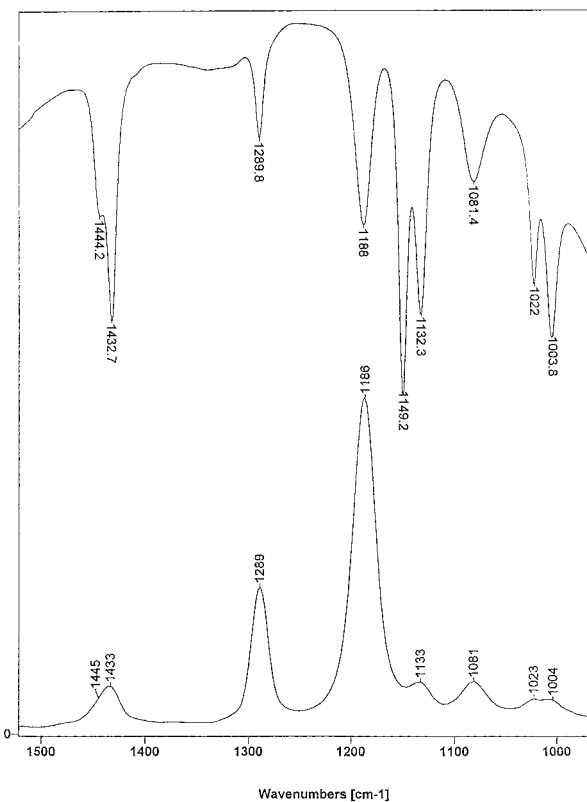


Figure 3. FTIR (top) and FT Raman (bottom) spectra of tetrazole (3) anion.

methods are longer than those predicted by the B3LYP and LDA methods, the calculated frequencies by the BLYP and BVWN methods are close to each other and most of the calculated frequencies by both methods are underestimated. The average error obtained by both methods is about twice that obtained by

the LDA method. This is compared to the lowest average error obtained by the BVWN method for furan and thiophene using the BS1 basis set.² Some calculations, as the BLYP/BS2, predict the C–H stretching frequencies in excellent agreement with the experimental frequencies as the predicted C–H bond length with BLYP/BS2 is close to that predicted by the LDA method.

Although the total average error by the MP2 method is bigger than that obtained by the DFT methods, the average error for the in-plane modes is smaller than that obtained by the BLYP and BVWN methods, as was also reported for furan and thiophene.² Some calculated frequencies by the MP2 method are underestimated as the MP2-predicted geometry is closer to the BLYP, and BVWN-predicted geometries than that predicted by the B3LYP and LDA methods. The calculated frequencies at the MP2/BS1 level in this work are very close to those predicted at the MP2/6-31G* level reported by Törnkvist et al.²⁴ The difference between the calculated frequencies for the C–H stretching modes by the above two methods is not more than 30 cm⁻¹ and for the other modes is not more than 4 cm⁻¹ except for ν_{12} where the difference is 11 cm⁻¹. Notice that the predicted geometries by both calculations are very close to each other with the exception of the predicted C–H bond length at the former level is shorter than that at the latter level by about 0.005 Å. The HF-calculated frequencies have the highest overestimation in accordance with the HF-predicted bond lengths are the shortest among the ab initio methods used in this work.

The effect of the basis set and inclusion of the diffuse functions, as was the case with the predicted geometries, is usually small with the calculated frequencies by the BS1 basis set have generally the lowest average error.

The calculated frequencies by the semiempirical methods are generally overestimated with the exception of the lower frequency in-plane modes which are underestimated. In general the calculated frequencies by the semiempirical methods are grossly overestimated by 100–250 cm⁻¹, with the calculated frequencies by the PM3 method have the lowest average error and those calculated by the MNDO method have the highest average error.

It is worth noting that the calculated frequencies by the B3LYP and MP2 methods and the corresponding experimental frequencies of $\nu_2/\nu_3/\nu_7$ for **3** are at 1296/1192/1169, 1279/1145/1234, and 1189/1188/1150 cm⁻¹, respectively, with the calculated frequency order assignment without observing the Raman polarization data would lead to a wrong assignment with the MP2 method, but not with the B3LYP method. This is also observed with the BLYP and BVWN methods but not with the LDA and HF methods.

Scale Factors. The calculated scale factors using 1SF scaling and the corresponding average errors are appended in Tables 4, 5, and 6 for **1**, **2**, and **3**, respectively. In the scaling step, the AS assignment of the studied molecules in Tables 4–6 was used.

A clear feature of the calculated scale factors for the three studied anions is that they are bigger than those reported for other neutral molecules^{2,8–10,14–16} but are close to those obtained by Brouwer using the UBLYP/6-31G* and UB3LYP/6-31G* methods. The calculated force field scale factors of furan and thiophene² at the B3LYP, LDA, BLYP, BVWN, HF, and MP2 levels using the BS1 basis set are 0.927, 0.963, 0.983, 0.965, 0.819, and 0.894, respectively. The calculated force field scale factors using 1SF reported by Rauhut et al.⁸ are 0.990 and 0.928 at the BLYP and B3LYP levels, respectively, using the 6-31G* basis set. The calculated frequency scale factors using 1SF

TABLE 4: Calculated Vibrational Frequencies (cm⁻¹) for 1^a

| symmetry | no. | B3LYP | | | | LDA | | | | exptl ^b | | |
|--------------------|-----------------------|-------|-------|-------|-------|-------|-------|-------|-------|--------------------|-------|-----------------|
| | | BS1 | BS2 | BS3 | BS4 | BS1 | BS2 | BS3 | BS4 | IR | Raman | SA ^c |
| A ₁ | 1 | 3198 | 3220 | 3179 | 3210 | 3139 | 3159 | 3126 | 3155 | | 3135 | 3135 |
| | 2 | 1473 | 1455 | 1456 | 1448 | 1458 | 1442 | 1443 | 1435 | 1439 | 1439 | 1439 |
| | 3 | 1233 | 1225 | 1229 | 1222 | 1247 | 1244 | 1247 | 1241 | | 1207 | 1207 |
| | 4 | 1163 | 1160 | 1169 | 1161 | 1175 | 1174 | 1182 | 1175 | 1146 | 1143 | 1146 |
| | 5 | 1106 | 1095 | 1089 | 1085 | 1083 | 1071 | 1068 | 1061 | 1097 | 1096 | 1097 |
| | 6 | 961 | 952 | 950 | 947 | 934 | 926 | 918 | 919 | 963 | 964 | 963 |
| B ₂ | 7 | 3170 | 3196 | 3150 | 3185 | 3112 | 3135 | 3098 | 3129 | | 3116 | 3116 |
| | 8 | 1436 | 1417 | 1416 | 1407 | 1408 | 1392 | 1392 | 1385 | 1412 | 1412 | 1412 |
| | 9 | 1156 | 1150 | 1138 | 1142 | 1166 | 1162 | 1160 | 1156 | 1230 ^d | | |
| | 10 | 1067 | 1058 | 1062 | 1054 | 1063 | 1059 | 1047 | 1049 | 1068 | | 1068 |
| | 11 | 967 | 962 | 967 | 960 | 939 | 934 | 939 | 934 | | 972 | 972 |
| A ₂ | 12 | 769 | 775 | 775 | 788 | 738 | 740 | 745 | 764 | | 725 | |
| | 13 | 671 | 668 | 678 | 676 | 664 | 658 | 672 | 670 | | 668 | |
| B ₁ | 14 | 770 | 757 | 763 | 755 | 748 | 733 | 746 | 734 | 800 ^e | | |
| | 15 | 734 | 730 | 736 | 732 | 726 | 723 | 722 | 724 | 773 ^e | | |
| error ^f | C-H | 59 | 83 | 39 | 72 | 4 | 22 | 14 | 17 | | | |
| | in-plane ^g | 15 | 11 | 12 | 12 | 22 | 25 | 29 | 29 | | | |
| | total | 24 | 25 | 17 | 24 | 18 | 24 | 26 | 27 | | | |
| SF ^h | | 0.965 | 0.962 | 0.978 | 0.964 | 0.995 | 0.991 | 1.006 | 0.996 | | | |
| error ^f | | 10 | 20 | 12 | 23 | 18 | 23 | 23 | 26 | | | |

| symmetry | no. | BLYP | | | | BVWN | | | | HF | | | |
|--------------------|------------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| | | BS1 | BS2 | BS3 | BS4 | BS1 | BS2 | BS3 | BS4 | BS1 | BS2 | BS3 | BS4 |
| A ₁ | 1 | 3099 | 3129 | 3079 | 3121 | 3131 | 3126 | 3113 | 3153 | 3373 | 3391 | 3362 | 3376 |
| | 2 | 1413 | 1395 | 1395 | 1387 | 1421 | 1405 | 1403 | 1398 | 1609 | 1593 | 1595 | 1586 |
| | 3 | 1164 | 1158 | 1162 | 1156 | 1162 | 1158 | 1160 | 1155 | 1357 | 1355 | 1362 | 1347 |
| | 4 | 1082 | 1080 | 1087 | 1080 | 1079 | 1079 | 1084 | 1079 | 1325 | 1318 | 1327 | 1315 |
| | 5 | 1068 | 1056 | 1048 | 1047 | 1076 | 1066 | 1057 | 1058 | 1194 | 1183 | 1181 | 1172 |
| | 6 | 909 | 900 | 900 | 896 | 914 | 907 | 907 | 904 | 1072 | 1065 | 1061 | 1056 |
| B ₂ | 7 | 3070 | 3104 | 3049 | 3096 | 3102 | 3136 | 3083 | 3128 | 3343 | 3365 | 3331 | 3348 |
| | 8 | 1370 | 1350 | 1349 | 1342 | 1380 | 1364 | 1360 | 1355 | 1593 | 1577 | 1578 | 1562 |
| | 9 | 1094 | 1091 | 1072 | 1082 | 1100 | 1099 | 1080 | 1090 | 1300 | 1285 | 1291 | 1272 |
| | 10 | 957 | 949 | 959 | 949 | 948 | 943 | 950 | 944 | 1226 | 1220 | 1211 | 1209 |
| | 11 | 932 | 926 | 931 | 925 | 937 | 933 | 936 | 932 | 1059 | 1055 | 1058 | 1049 |
| A ₂ | 12 | 725 | 726 | 732 | 742 | 734 | 737 | 744 | 750 | 904 | 914 | 898 | 902 |
| | 13 | 636 | 637 | 647 | 649 | 640 | 640 | 649 | 651 | 747 | 740 | 747 | 737 |
| B ₁ | 14 | 738 | 722 | 732 | 721 | 745 | 732 | 740 | 731 | 865 | 856 | 854 | 848 |
| | 15 | 699 | 694 | 699 | 697 | 700 | 696 | 701 | 699 | 823 | 816 | 823 | 812 |
| error ^f | C-H | 41 | 9 | 62 | 17 | 9 | 15 | 28 | 15 | 233 | 253 | 221 | 237 |
| | in-planes ^g | 51 | 61 | 59 | 65 | 48 | 56 | 56 | 60 | 141 | 133 | 134 | 124 |
| | total | 49 | 51 | 60 | 55 | 40 | 48 | 50 | 51 | 159 | 157 | 151 | 147 |
| SF ^h | | 1.044 | 1.035 | 1.059 | 1.041 | 1.027 | 1.027 | 1.026 | 1.026 | 0.834 | 0.833 | 0.842 | 0.842 |
| error ^f | | 33 | 42 | 27 | 44 | 33 | 33 | 31 | 31 | 37 | 29 | 35 | 28 |

| symmetry | no. | MP2 | | | | AM1 | PM3 | MNDO | AD ⁱ |
|--------------------|-----------------------|-------|-------|-------|-------|-------|-------|-------|--------------------------------|
| | | BS1 | BS2 | BS3 | BS4 | | | | |
| A ₁ | 1 | 3287 | 3298 | 3242 | 3255 | 3295 | 3160 | 3496 | C-H str ^j |
| | 2 | 1526 | 1500 | 1500 | 1477 | 1560 | 1556 | 1657 | C-C str + C-H def ^k |
| | 3 | 1270 | 1259 | 1261 | 1238 | 1476 | 1476 | 1518 | C-N str |
| | 4 | 1137 | 1125 | 1126 | 1097 | 1407 | 1295 | 1468 | N-N str + ring def |
| | 5 | 1112 | 1094 | 1095 | 1075 | 1117 | 1047 | 1179 | C-C str + C-H def + ring def |
| | 6 | 942 | 926 | 929 | 905 | 943 | 893 | 980 | ring def + C-H def |
| B ₂ | 7 | 3259 | 3272 | 3213 | 3228 | 3286 | 3145 | 3483 | C-H str |
| | 8 | 1444 | 1422 | 1414 | 1388 | 1590 | 1510 | 1656 | C-N str + C-H def |
| | 9 | 1214 | 1202 | 1197 | 1174 | 1445 | 1251 | 1517 | C-N str + C-H def |
| | 10 | 1075 | 1056 | 1064 | 1033 | 1140 | 1057 | 1219 | N-N str |
| | 11 | 958 | 948 | 950 | 934 | 952 | 894 | 921 | ring def |
| A ₂ | 12 | 737 | 697 | 757 | 759 | 844 | 831 | 919 | C-H wag ^l |
| | 13 | 628 | 547 | 658 | 655 | 608 | 567 | 581 | ring torsion |
| B ₁ | 14 | 763 | 729 | 761 | 752 | 821 | 764 | 875 | C-H wag |
| | 15 | 722 | 700 | 726 | 710 | 720 | 628 | 674 | ring torsion |
| error ^f | C-H | 148 | 160 | 102 | 116 | 165 | 27 | 364 | |
| | in-plane ^g | 31 | 28 | 25 | 37 | 120 | 99 | 175 | |
| | total | 54 | 54 | 40 | 53 | 129 | 84 | 212 | |
| SF ^h | | 0.924 | 0.928 | 0.951 | 0.957 | 0.858 | 0.939 | 0.773 | |
| error ^f | | 34 | 42 | 34 | 47 | 79 | 86 | 78 | |

^a See footnote *b* in Table 1. ^b Reference 24. ^c Assignment used in scaling. ^d Assignment as fundamental is questionable; see text. ^e KBr Pellet. ^f Average error in cm⁻¹. ^g Excluding C-H stretching modes. ^h Scale factor. ⁱ Approximate description based on B3LYP/6-31G** force field. Only values greater than 10% are considered. ^j str = stretching. ^k def = deformation. ^l wag = wagging.

scaling reported by Scott et al.⁹ are 0.8992, 0.9370, 0.9945, and 0.9614 at the HF/6-31G**, MP2/6-31G*, BLYP/6-31G* and

B3LYP/6-31G* levels, respectively. These correspond to 0.8086, 0.8780, 0.9890, and 0.9243, respectively, for force field

TABLE 5: Calculated Vibrational Frequencies (cm⁻¹) for 2^a

| symmetry | no. | B3LYP | | LDA | | BLYP | | BVWN | | exptl | | |
|--------------------|---------------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|--------------------|-----------------|
| | | BS1 | BS2 | BS1 | BS2 | BS1 | BS2 | BS1 | BS2 | IR | Raman ^b | SA ^c |
| A ₁ | 1 | 3157 | 3191 | 3096 | 3126 | 3061 | 3102 | 3095 | 3135 | 3114 | | |
| | 2 | 1524 | 1499 | 1497 | 1474 | 1453 | 1426 | 1465 | 1442 | 1490 | 1491(p,0.22) | 1490 |
| | 3 | 1255 | 1254 | 1255 | 1256 | 1188 | 1188 | 1189 | 1190 | | 1257(p,0.05) | 1257 |
| | 4 | 1175 | 1166 | 1174 | 1169 | 1126 | 1116 | 1133 | 1124 | 1148 | 1150(p,0.15) | 1148 |
| | 5 | 1042 | 1041 | 1071 | 1066 | 958 | 961 | 957 | 958 | 1029 | 1030(p,0.12) | 1029 |
| | 6 | 985 | 975 | 961 | 952 | 943 | 930 | 938 | 931 | 976 | 977(p,0.36) | 976 |
| B ₂ | 7 | 3149 | 3186 | 3089 | 3121 | 3053 | 3096 | 3087 | 3130 | 3103 | | |
| | 8 | 1407 | 1389 | 1393 | 1377 | 1339 | 1320 | 1350 | 1334 | 1389 | 1388(dp,0.65) | 1389 |
| | 9 | 1265 | 1270 | 1285 | 1291 | 1197 | 1203 | 1195 | 1202 | 1258 | | 1258 |
| | 10 | 1214 | 1207 | 1177 | 1170 | 1163 | 1156 | 1172 | 1167 | 1194 | | 1194 |
| | 11 | 990 | 980 | 971 | 961 | 951 | 940 | 955 | 946 | 992 | | 992 |
| A ₂ | 12 | 826 | 823 | 790 | 784 | 780 | 774 | 789 | 787 | | | |
| B ₁ | 13 | 710 | 704 | 709 | 701 | 679 | 672 | 679 | 674 | | | |
| | 14 | 826 | 826 | 787 | 788 | 780 | 779 | 791 | 791 | | | |
| error ^d | 15 | 706 | 696 | 699 | 688 | 679 | 669 | 679 | 672 | | 964 | |
| | C-H in-plane ^e | | | | | | | | | | | |
| | total | 15 | 9 | 18 | 22 | 46 | 55 | 42 | 49 | | | |
| SF ^f | | 0.973 | 0.988 | 0.986 | 0.999 | 1.075 | 1.094 | 1.068 | 1.082 | | | |
| error ^d | | 9 | 8 | 18 | 22 | 21 | 11 | 19 | 14 | | | |

| symmetry | no. | HF | | MP2 | | AM1 | PM3 | MNDO | AD ^g |
|--------------------|---------------------------|-------|-------|-------|-------|-------|-------|-------|--------------------------------|
| | | BS1 | BS2 | BS1 | BS2 | | | | |
| A ₁ | 1 | 3333 | 3361 | 3251 | 3275 | 3252 | 3112 | 3486 | C-H str ^h |
| | 2 | 1699 | 1675 | 1534 | 1500 | 1672 | 1548 | 1731 | C-H def ⁱ + C-N str |
| | 3 | 1403 | 1396 | 1250 | 1244 | 1506 | 1435 | 1563 | C-N str |
| | 4 | 1276 | 1267 | 1206 | 1191 | 1405 | 1232 | 1495 | C-H def + C-N str |
| | 5 | 1171 | 1170 | 1071 | 1055 | 1109 | 1001 | 1197 | N-N str |
| | 6 | 1082 | 1073 | 971 | 959 | 934 | 855 | 928 | ring def |
| B ₂ | 7 | 3324 | 3355 | 3244 | 3269 | 3244 | 3105 | 3481 | C-H str |
| | 8 | 1570 | 1553 | 1449 | 1418 | 1456 | 1458 | 1538 | C-N str + C-H def |
| | 9 | 1394 | 1392 | 1298 | 1292 | 1411 | 1356 | 1503 | C-N str |
| | 10 | 1319 | 1313 | 1221 | 1211 | 1163 | 1060 | 1265 | C-H def + C-N str |
| | 11 | 1084 | 1076 | 983 | 968 | 961 | 889 | 967 | ring def |
| A ₂ | 12 | 973 | 973 | 789 | 738 | 839 | 783 | 916 | C-H wag |
| | 13 | 782 | 775 | 702 | 667 | 682 | 603 | 641 | ring torsion |
| B ₁ | 14 | 972 | 971 | 799 | 771 | 851 | 785 | 908 | C-H wag |
| | 15 | 767 | 758 | 702 | 681 | 626 | 567 | 590 | ring torsion |
| error ^d | C-H in-plane ^e | | | | | | | | |
| | total | 140 | 131 | 32 | 24 | 121 | 97 | 178 | |
| SF ^f | | 0.796 | 0.808 | 0.949 | 0.975 | 0.818 | 0.933 | 0.738 | |
| error ^d | | 15 | 12 | 21 | 20 | 85 | 85 | 86 | |

^a See footnote *b* in Table 1. ^b Polarization and polarization ratio between parenthesis. p: polarized. dp: depolarized. ^{c, d, e, f, g} See footnotes *c, f, g, h, and i*, respectively, in Table 4. ^h str = stretch. ⁱ def = deformation. ^j wag = wagging.

scaling. Those reported by Wong¹⁰ using 1SF frequency scaling are 0.9427, 0.9833, 0.9940, 0.9820, and 0.9613 at the MP2, LDA, BLYP, BVWN, and B3LYP levels, respectively, using the 6-31G* basis set. These correspond to force field scale factors of 0.8887, 0.9669, 0.9880, 0.9643, and 0.9241, respectively. These scale factors are quite close to those obtained by us for neutral five-membered heterocyclic molecules^{2,14-16} but smaller than those calculated for studied anions. Those reported by Brouwer for *N,N,N',N'*-tetramethyl-*p*-phenylenediamine radical cation using frequency scaling are 0.982 and 1.020 by the UBLYP and UB3LYP methods, respectively. These correspond to force field scale factors of 0.964 and 1.040, respectively. This is in agreement with our observation that the scale factors of the studied molecules are bigger than those of neutral molecules.

The BLYP and BVWN scale factors are bigger than 1.0 since all the calculated frequencies by both methods are underestimated and both have the longest bond lengths among the DFT methods used. The LDA scale factors are close to 1.0 since some of the calculated frequencies are underestimated and some are overestimated. The B3LYP scale factors are less than 1.0

since most of the calculated frequencies are overestimated as the B3LYP method has the shortest predicted bond lengths. The calculated scale factors by the B3LYP and LDA methods for the studied anions are generally less divergent than those obtained by the other methods. The effect of the basis set and inclusion of diffuse functions, as was observed for the optimized geometries and calculated frequencies, is small. The largest difference is found for the MP2 method. For the semiempirical methods, the PM3 scale factors are the biggest and the MNDO scale factors are the smallest since the PM3 had the smallest average error and the MNDO had the biggest average error. However, the PM3 scale factors are the least divergent and the average error obtained by the three semiempirical methods is very close to each other.

As was observed for furan and thiophene,² the B3LYP average error is the lowest followed by the LDA average error, although for **3** this order is reversed. The semiempirical methods have the highest average error, about 90 cm⁻¹. The BS1 and BS3 have average errors close to each other, but the addition of the diffuse functions, BS2 and BS4, is accompanied by bigger average errors except at the HF level.

TABLE 6: Calculated Vibrational Frequencies (cm⁻¹) for 3^a

| symmetry | no. | B3LYP | | LDA | | BLYP | | BVWN | | exptl | | |
|--------------------|-----------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|--------------------|-----------------|
| | | BS1 | BS2 | BS1 | BS2 | BS1 | BS2 | BS1 | BS2 | IR | Raman ^b | SA ^c |
| A ₁ | 1 | 3199 | 3228 | 3139 | 3165 | 3101 | 3137 | 3135 | 3171 | 3114 | 3136 | 3114 |
| | 2 | 1296 | 1297 | 1294 | 1298 | 1219 | 1222 | 1220 | 1224 | 1289 | 1290(p,0.03) | 1289 |
| | 3 | 1192 | 1189 | 1192 | 1191 | 1099 | 1096 | 1096 | 1095 | 1188 | 1186(p,0.05) | 1188 |
| | 4 | 1075 | 1063 | 1111 | 1105 | 992 | 977 | 993 | 982 | 1081 | 1081(p,0.13) | 1081 |
| | 5 | 1022 | 1018 | 999 | 995 | 939 | 940 | 931 | 932 | 1022 | 1023(p) | 1022 |
| B ₂ | 6 | 1473 | 1457 | 1452 | 1438 | 1398 | 1380 | 1408 | 1393 | 1433 | 1433(dp,0.72) | 1433 |
| | 7 | 1169 | 1166 | 1169 | 1171 | 1120 | 1114 | 1129 | 1124 | 1149 | | 1149 |
| | 8 | 1143 | 1136 | 1131 | 1120 | 1070 | 1065 | 1071 | 1066 | 1132 | 1133(dp,0.73) | 1132 |
| | 9 | 1009 | 998 | 1005 | 994 | 934 | 923 | 929 | 921 | 1004 | 1004(dp,0.65) | 1004 |
| A ₂ | 10 | 754 | 746 | 752 | 743 | 717 | 709 | 717 | 711 | | 760 | |
| B ₁ | 11 | 830 | 832 | 792 | 794 | 784 | 786 | 796 | 798 | | | |
| | 12 | 722 | 715 | 717 | 710 | 690 | 684 | 691 | 687 | | 697 | |
| error ^d | C-H | 85 | 114 | 25 | 51 | 13 | 23 | 21 | 57 | | | |
| | in-plane ^e | 12 | 11 | 12 | 13 | 67 | 73 | 66 | 71 | | | |
| | total | 20 | 22 | 14 | 18 | 61 | 68 | 61 | 69 | | | |
| SF ^f | | 0.962 | 0.959 | 0.983 | 0.980 | 1.057 | 1.050 | 1.043 | 1.034 | | | |
| error ^d | | 17 | 23 | 11 | 16 | 41 | 51 | 49 | 58 | | | |

| symmetry | no. | HF | | MP2 | | AM1 | PM3 | MNDO | AD ^g |
|--------------------|-----------------------|-------|-------|-------|-------|-------|-------|-------|------------------------------|
| | | BS1 | BS2 | BS1 | BS2 | | | | |
| A ₁ | 1 | 3380 | 3400 | 3290 | 3308 | 3265 | 3123 | 3507 | C-H str ^b |
| | 2 | 1484 | 1478 | 1279 | 1274 | 1533 | 1477 | 1651 | C-N str |
| | 3 | 1396 | 1389 | 1145 | 1123 | 1479 | 1445 | 1568 | N-N str |
| | 4 | 1252 | 1239 | 1109 | 1094 | 1409 | 1210 | 1515 | N-N str |
| | 5 | 1133 | 1127 | 1005 | 992 | 951 | 891 | 986 | ring def |
| B ₂ | 6 | 1648 | 1632 | 1499 | 1473 | 1647 | 1526 | 1724 | C-H def + C-N str |
| | 7 | 1296 | 1295 | 1234 | 1224 | 1391 | 1248 | 1476 | C-H def + C-N str |
| | 8 | 1273 | 1265 | 1145 | 1128 | 1136 | 1042 | 1222 | ring def + N-N str |
| | 9 | 1141 | 1129 | 1001 | 981 | 938 | 888 | 1015 | ring def + N-N str + C-H def |
| A ₂ | 10 | 842 | 836 | 743 | 714 | 753 | 640 | 707 | ring torsion + C-H wag |
| B ₁ | 11 | 974 | 970 | 810 | 786 | 852 | 796 | 918 | C-H wag + ring torsion |
| | 12 | 796 | 787 | 711 | 691 | 685 | 609 | 647 | ring torsion + C-H wag |
| error ^d | C-H | 286 | 176 | 194 | 151 | 9 | 393 | | |
| | in-plane ^e | 165 | 156 | 32 | 32 | 182 | 137 | 241 | |
| | total | 176 | 139 | 48 | 50 | 178 | 123 | 258 | |
| SF ^f | | 0.797 | 0.800 | 0.929 | 0.934 | 0.798 | 0.907 | 0.699 | |
| error ^d | | 35 | 27 | 43 | 52 | 128 | 115 | 125 | |

^a See footnote *b* in Table 1. ^b See footnote *b* in Table 5. ^c *d*, *e*, *f*, *g*, *h*, and *i*, respectively, in Table 4. ^h str = stretch. ⁱ def = deformation. ^j wag = wagging.

TABLE 7: Calculated IR Absorption Intensities (km/mol) for 1 Using the 6-31G** Basis Set

| symmetry | no. | B3LYP | LDA | BLYP | BVWN | HF | MP2 | AM1 | PM3 | MNDO | exptl |
|----------------|-----|-------|------|------|------|------|------|------|------|------|----------------|
| A ₁ | 1 | 73.6 | 58.9 | 90.3 | 99.0 | 69.0 | 58.2 | 12.0 | 13.1 | 0.8 | |
| | 2 | 0.3 | 0.4 | 0.5 | 0.5 | 0.1 | 0.3 | 7.1 | 4.4 | 6.7 | m ^b |
| | 3 | 0.5 | 0.5 | 0.3 | 0.3 | 5.5 | 0.0 | 0.1 | 23.3 | 1.4 | |
| | 4 | 7.8 | 6.7 | 6.5 | 2.3 | 9.2 | 0.3 | 2.5 | 13.7 | 15.0 | s |
| | 5 | 4.4 | 4.8 | 3.7 | 8.2 | 3.3 | 8.2 | 3.0 | 2.5 | 0.3 | m |
| B ₂ | 6 | 1.8 | 2.1 | 1.4 | 1.6 | 2.0 | 1.4 | 0.5 | 0.8 | 3.9 | s |
| | 7 | 54.6 | 40.9 | 67.6 | 73.7 | 52.5 | 42.4 | 9.0 | 11.6 | 0.7 | |
| | 8 | 9.0 | 9.3 | 8.3 | 8.1 | 11.5 | 3.6 | 11.8 | 48.0 | 16.9 | m |
| | 9 | 0.5 | 2.1 | 4.2 | 5.6 | 5.6 | 0.4 | 0.9 | 36.8 | 1.0 | w |
| | 10 | 28.5 | 24.7 | 17.7 | 11.1 | 25.7 | 30.4 | 7.0 | 10.4 | 1.9 | vs |
| | 11 | 16.3 | 18.0 | 21.8 | 27.9 | 18.0 | 12.4 | 4.2 | 4.4 | 4.2 | |
| A ₂ | 12 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | |
| | 13 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | |
| B ₁ | 14 | 22.5 | 15.2 | 17.8 | 19.1 | 36.3 | 26.2 | 77.9 | 58.6 | 28.4 | |
| | 15 | 0.8 | 9.7 | 1.2 | 0.7 | 0.4 | 1.8 | 2.4 | 2.9 | 1.1 | |

^a Reference 24. ^b vs: very strong. s: strong. m: medium. w: weak. vw: very weak.

IR Absorption Intensities. The calculated IR absorption intensities of **1**, **2**, and **3** are given in Tables 7, 8, and 9, respectively, using the BS1 basis set only as the calculated IR absorption intensities showed little dependence on the basis set used. Except with the semiempirical methods, good qualitative agreement between the calculated and experimental intensities is found but the agreement is not as excellent as previously reported.^{1,2,3,8,15,16,40} For example, ν_2 and ν_6 of **1**, ν_3 of **2**, and ν_2 of **3** are not predicted to be as intense as observed

experimentally. The agreement with the semiempirical methods is generally less satisfactory.

Bands corresponding to the C-H stretching modes are predicted by all methods used, except by the semiempirical methods, to be very intense while most of these bands are too weak to be observed experimentally. This is contrary to what was observed for neutral five-membered heterocyclic molecules.^{2,14-16} This was also observed by Langhoff for polycyclic aromatic hydrocarbon neutrals, cations, and anions.²¹

TABLE 8: Calculated IR Absorption Intensities (km/mol) for 2 Using the 6-31G Basis Set**

| symmetry | no. | B3LYP | LDA | BLYP | BVWN | HF | MP2 | AM1 | PM3 | MNDO | exptl ^a |
|----------------|-----|-------|-------|-------|-------|-------|-------|------|-------|------|--------------------|
| A ₁ | 1 | 9.9 | 7.6 | 12.3 | 13.6 | 9.2 | 8.4 | 3.9 | 4.1 | 0.2 | |
| | 2 | 41.5 | 40.7 | 35.5 | 35.5 | 63.1 | 31.4 | 34.5 | 118.6 | 53.3 | vs ^b |
| | 3 | 0.3 | 0.0 | 0.2 | 0.5 | 0.6 | 2.5 | 1.1 | 0.0 | 0.7 | |
| | 4 | 18.8 | 4.0 | 20.8 | 24.4 | 21.1 | 12.7 | 3.8 | 11.9 | 1.2 | vs |
| | 5 | 17.0 | 27.4 | 2.2 | 0.6 | 33.4 | 15.8 | 14.3 | 18.9 | 8.1 | m |
| | 6 | 10.8 | 8.4 | 18.8 | 19.9 | 11.4 | 8.1 | 3.7 | 7.2 | 2.4 | m |
| B ₂ | 7 | 170.8 | 137.9 | 196.0 | 210.5 | 180.1 | 139.7 | 34.5 | 29.7 | 0.0 | |
| | 8 | 4.8 | 7.5 | 4.7 | 4.2 | 0.8 | 14.8 | 0.8 | 3.5 | 1.4 | vw |
| | 9 | 18.7 | 12.7 | 15.3 | 15.5 | 35.5 | 11.0 | 1.5 | 9.0 | 0.1 | s |
| | 10 | 0.9 | 0.0 | 0.8 | 1.8 | 7.4 | 0.2 | 0.1 | 0.5 | 0.1 | m |
| | 11 | 9.9 | 11.7 | 9.9 | 9.3 | 10.3 | 9.7 | 6.1 | 6.9 | 5.5 | w |
| A ₂ | 12 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | |
| | 13 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | |
| B ₁ | 14 | 7.7 | 8.5 | 7.9 | 8.2 | 5.5 | 15.2 | 75.4 | 43.6 | 22.8 | |
| | 15 | 18.8 | 18.3 | 14.2 | 14.9 | 34.7 | 16.8 | 28.7 | 23.3 | 19.3 | |

^a This work. ^b See footnotes *b* in Table 7.

TABLE 9: Calculated IR Absorption Intensities (km/mol) for 3 Using the 6-31G Basis Set**

| symmetry | no. | B3LYP | LDA | BLYP | BVWN | HF | MP2 | AM1 | PM3 | MNDO | exptl ^a |
|----------------|-----|-------|------|------|------|------|------|------|------|------|--------------------|
| A ₁ | 1 | 56.5 | 42.7 | 69.4 | 75.7 | 57.3 | 45.3 | 28.8 | 26.8 | 2.6 | |
| | 2 | 0.3 | 0.2 | 0.2 | 0.2 | 0.0 | 1.5 | 4.7 | 10.9 | 1.8 | m ^b |
| | 3 | 8.1 | 5.5 | 6.3 | 6.8 | 16.8 | 5.9 | 12.0 | 17.1 | 15.4 | s |
| | 4 | 4.2 | 5.8 | 1.3 | 2.1 | 5.8 | 10.3 | 0.6 | 8.5 | 14.9 | m |
| | 5 | 12.5 | 11.5 | 15.6 | 14.7 | 10.2 | 6.5 | 2.7 | 2.6 | 2.9 | w |
| B ₂ | 6 | 17.4 | 18.3 | 14.5 | 14.1 | 26.7 | 13.2 | 19.8 | 85.1 | 30.7 | vs |
| | 7 | 6.3 | 2.7 | 11.6 | 13.6 | 11.4 | 0.6 | 3.5 | 19.0 | 1.9 | vs |
| | 8 | 14.8 | 17.1 | 5.2 | 4.4 | 21.0 | 18.2 | 7.8 | 9.2 | 4.1 | vs |
| | 9 | 15.9 | 14.4 | 17.9 | 18.5 | 13.3 | 10.2 | 2.3 | 5.6 | 0.8 | w |
| A ₂ | 10 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | |
| B ₁ | 11 | 6.4 | 7.2 | 6.3 | 6.3 | 5.4 | 10.2 | 48.4 | 27.2 | 17.1 | |
| | 12 | 6.8 | 6.6 | 5.1 | 5.4 | 13.1 | 5.3 | 10.8 | 7.5 | 7.1 | |

^a This work. ^b See footnotes *b* in Table 7.

It is interesting to notice the similarity between the experimental IR and Raman spectra of **2** and **3**. For example, the pattern of bands in the Raman spectra of **2/3** at 1491/1432, 1388/1289, 1257/1187, 1150/1133, 1030/1022, and 977/1005 cm⁻¹ is quite similar and of comparable intensity. This pattern is well reproduced by the calculated IR spectra of **2** and **3** except ν_{10} of **2** and ν_7 of **3** (both correspond to C–H deformation + C–N stretching modes) are predicted to be weaker than observed experimentally. The spectra of **1**, especially the IR spectra, is somewhat different from those of the other two molecules.

Conclusion

In the present publication, the vibrational spectra of **2** and **3** were presented. A comparison between the optimized geometries, vibrational frequencies, scale factors, and IR absorption intensities for the three anions, **1**, **2**, and **3**, was made. The calculations were performed at the same levels of theory as those reported previously for furan and thiophene,² B3LYP, BLYP LDA, BVWN, HF, and MP2 using basis sets without diffuse functions, BS1 and BS3, and with diffuse functions, BS2 and BS4. The calculated results were compared with the experimental data reported in this work for **2** and **3** and available in the literature for **1**. An important conclusion is that the scale factors obtained for the studied anions are bigger than those obtained for neutral molecules and are similar to those obtained for a radical cation using the UBLYP and UB3LYP methods.²² Since scale factors for common use are calculated as an average scale factors for a large number of molecules^{8–10} rather than for single molecules, this conclusion indicate that ionic species require a unique scale factor separate from those calculated for neutral molecules.

The reported experimental spectra of the studied anions in the reported in this work were measured for the solution phase

rather than the gas phase since measurement of the gas-phase spectrum of most anions is not practical. It is not clear how the interaction of the anions with the solvent molecules or the cation may effect the results. For this reason, the calculations were extended to include the 1,2,3-triazole–Na⁺ complex. The calculations of the latter were done at the same levels as for **1** using the BS1 and BS2 basis sets. Although these results are not presented in the current publication for space reasons, the calculated results were very similar to those obtained for **1**, except with the BS2 basis set the Na–N₂ bond length was shorter by about 0.04 Å and the Na–N₂–N₁ bond angle was bigger by about 5° with the B3LYP method. The calculated frequencies for the modes corresponding to the Na⁺ by the B3LYP BS1/BS2 basis sets are 349/355, 125/53, and 122/51 cm⁻¹. This indicates that the diffuse functions have big effect on the results related to the cation part. There is no experimental data available in this far-IR region to judge these results.

The results were related as much as possible to the optimized geometries. Rationalization of the behavior of different density functionals is not possible since these functionals were optimized with respect to thermodynamic properties rather than with respect to vibrational frequencies. However, it is clear that the three-parameter B3LYP functional with the HF component outperforms the other functionals. The performance of the semiempirical methods described the modes best among all methods used.

Acknowledgment. A.A.E. thanks P. Pulay visiting Professor at Universität Stuttgart and T. A. Keiderling of University of Illinois at Chicago for their support and helpful discussions. A generous grant of computer time at the Centro Svizzero di Calcolo Scientifico (CSCS) in Manno is gratefully acknowledged.

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